THERMAL PROPERTIES OF MATTER

HEAT AND TEMPERATURE :-

→ Heat and work both are ways of transfer of energy.
Heat → Non-mechanical Way
work → Mechanical Way

→ Temperature is the measurement of hotness and coldness.
Reason : नहीं है।
Feel : Avg. kinetic energy of each molecule
(Energy per molecule)

→ Heat is flown due to temp. difference. i.e. difference in vibrational kinetic energy.

→ When two bodies at different temp. are connected the molecules with greater vibrational energy gives energy to molecules with lower energy and hence temp. of hotter body falls and that of colder body rises up to same value.

THERMOMETRY :- Measure of temperature.

→ For any scale...
$$\frac{\text{Reading} - \text{ice point}}{\text{Steam point} - \text{ice point}} = \text{Constant}$$

(Reference Points)	Celcius	Fahrenheit	Kelvin
Boiling of water (Steam Point)	100°C	212 °F	373.15 K
Melting of ice (Ice point)	0 °C	32 °F	273.15 K

→ For fahrenheit and celcius...

$$\frac{F-32}{212-32} = \frac{C-0}{100-0} \Rightarrow \boxed{\frac{C}{5} = \frac{F-32}{9}}$$

→ For kelvin and celcius...

$$\frac{K-273.15}{373.15-273.15} = \frac{C-0}{100-0} \Rightarrow \boxed{K = C + 273.15}$$

→ General form...

$$\frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15} = \frac{C-0}{100-0}$$

Q. Convert 27°C in K and F.

Sol.
$$\frac{27-0}{100-0} = \frac{K-273.15}{373.15-273.15} = \frac{F-32}{212-32}$$

Find the value of F and K.

Q. For Reumer's scale ice point is 0°R and steam point is 80°R. Find the value of 20°C in terms of °R.

Sol.
$$\frac{C-0}{100-0} = \frac{R-0}{80-0} \Rightarrow \frac{C}{100} = \frac{R}{80} \Rightarrow R = \frac{80}{100} \times 20 = 16\,°R.$$

Q. If temp. is changed from 30°C to 60°C. Find the value of ΔT in F and K.

Sol.

$$\left.\begin{array}{l} 0°C \rightarrow 100°C \\ 273.15\ K \rightarrow 373.15\ K \end{array}\right\}$$ Change is same

$$\Rightarrow \Delta T_c = \Delta T_K = 30°C$$

$$\left.\begin{array}{l} 0°C \rightarrow 100°C \\ 32\ F \rightarrow 212\ F \end{array}\right\}$$ $100\ \Delta T_c \rightarrow 180\ \Delta T_F$

$$\Rightarrow \Delta T_c = 1.8\,\Delta T_F$$

Now... $\cancel{\Delta T_F = \frac{\Delta T_c}{1.8}}$ Jab °C me 1 का change हो °F में 1.8 का change होता है।

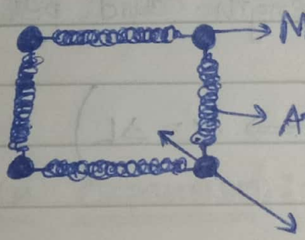
$\Rightarrow \Delta T_F = 1.8 \times \Delta T_c = 1.8 \times 30 = 54\ °F$

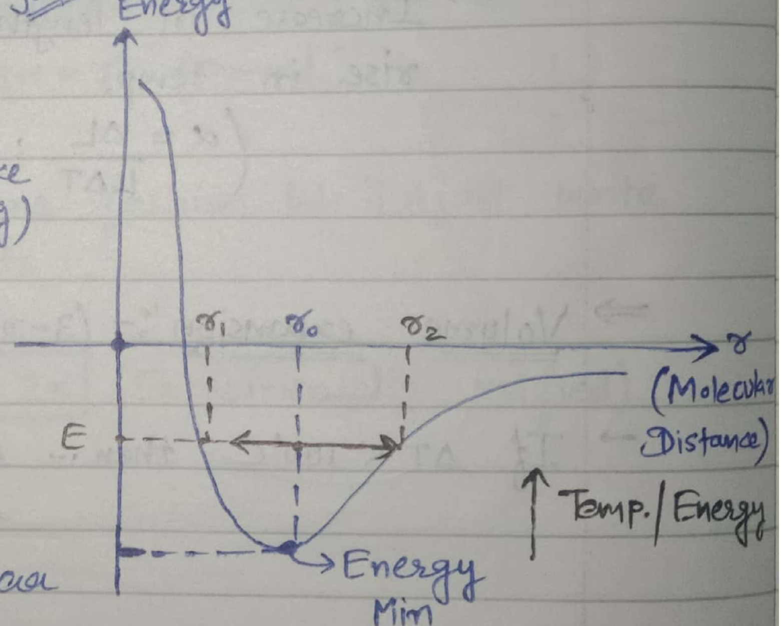
→ Hence we may conclude that...

$$\boxed{\Delta C = \Delta K = \frac{\Delta F}{1.8}}$$ 100%

THERMAL EXPANSION :-

→ When we give heat to a substance; its temp. rises and it expands.





Heat given ⇒ Molecules vibrate

⇒ Expansion

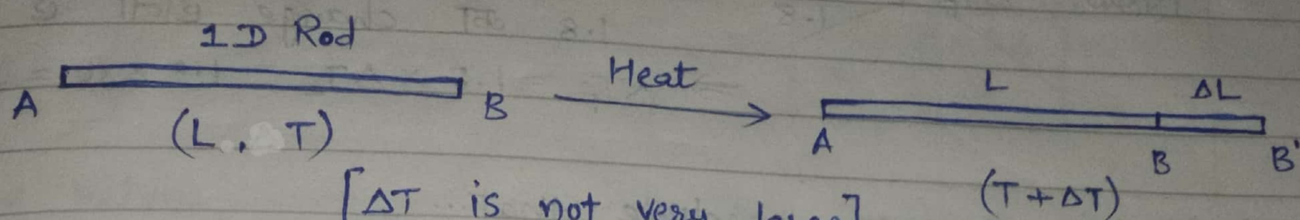
Sawal Utha → Vibrate karenge to door hi kyu jayenge... pass bhi aa sakte hai.

⇒ Avg. position same

⇒ No expansion ✗

⇒ Main reason of expansion :- The well of energy-distance is not symmetrical and hence avg. distance is increases and hence substance expands.

⇒ Linear expansion :-



[ΔT is not very large]

It is observed that... $\underbrace{\Delta L \propto \Delta T \text{ and } \Delta L \propto L}$

$$\boxed{\Delta L = L\alpha \Delta T}$$

Co-efficient of liner expansion

$\boxed{\alpha}$ → depends upon material
→ depends upon temp. (अपने लिए constant)
→ Increase in length per unit length and per unit rise in temp.

$$\left(\alpha = \frac{\Delta L}{L\Delta T} \; ; \text{if } \begin{array}{l} L = 1\text{ m} \\ \Delta T = 1 \text{ unit} \end{array} \Rightarrow \alpha = \Delta L\right)$$

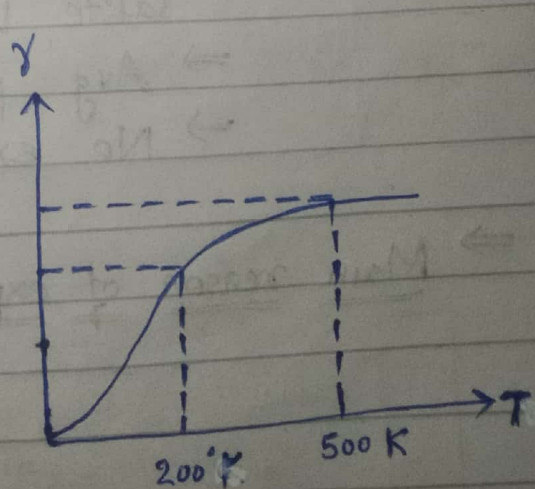
⇒ Volume expansion :- (3-D)

→ If $\Delta T < 100^\circ C$ then ... $\left.\begin{array}{l}\Delta V \propto V \\ \Delta V \propto \Delta T\end{array}\right\}$ $\boxed{\Delta V = V\gamma \Delta T}$

Co-efficient of volume expansion

$\boxed{\gamma}$ → depends on material
→ depends on temp. largely
→ Increase in volume per unit volume and per unit rise in temp...

$$\left(\gamma = \frac{\Delta V}{V\Delta T} \; ; \begin{array}{l} V = 1\text{ m}^3 \\ \Delta T = 1 \text{ unit} \end{array} \Rightarrow \gamma = \Delta V\right)$$

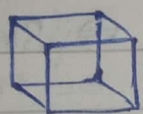


→ From graph we may conclude that...
i) At lower temp... $\gamma \uparrow$ with temp. $\uparrow$
ii) At higher temp... γ is nearly constant

⇒ **Isotropic Solid** :- (Amorphus)
Same properties in all directions (Disordered)

→ For these type of solids... $\boxed{\gamma = 3\alpha}$ (Experimentally varified)

Proof:



(V, l)

$V = l^3 \Rightarrow dV = 3l^2 \cdot dl$

Rising temp. by dT...

$dV = V\gamma\Delta T$

$\Rightarrow 3l^2 \cdot dl = V\gamma\Delta T$

$\Rightarrow 3l^2(l\alpha\Delta T) = l^3\gamma\Delta T$

$\Rightarrow 3\alpha = \gamma$. (proved.)

⇒ **Area expansion** :- All is same fashion. We'll just write the results.

$$\boxed{\Delta A = A\beta\Delta T}$$

β → Co-efficient of superficial/Area expansion.

$\boxed{\beta = 2\alpha}$ (Experimentally verified)

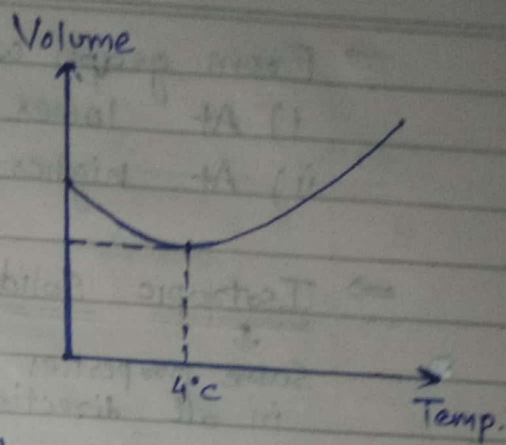
Q. Find the value of γ for an ideal gas at constant pres

Sol. $PV = nRT \Rightarrow PdV = nRdT$

$\Rightarrow P(V\gamma dT) = nRdT$

$\Rightarrow \gamma = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$

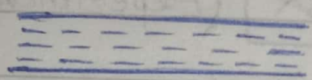
ANAMOLOUS EXPANSION OF WATER :-



→ 4°c से नीचे ⇒ contract होता है।
4°c से उपर ⇒ expand होता है।

⇒ **Important Observations :-**

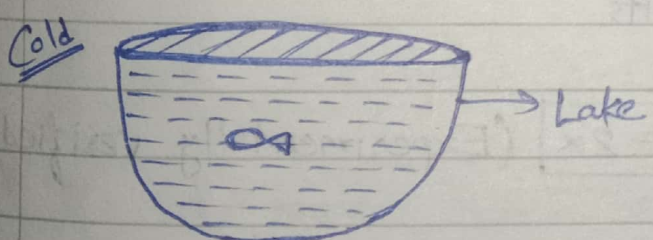
1.) In cold countries water pipes are burst.



As temp ↓↓ below 4°c...
Water expands. ⇒ Burst

2.) At cold nights crops are destroyed. B'coz फूल की डाली burst हो गई।

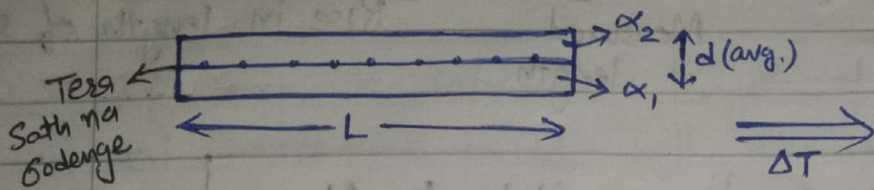
3.) Ice freezes only at lake surfaces and hence aquatic creatures can leave below surface.

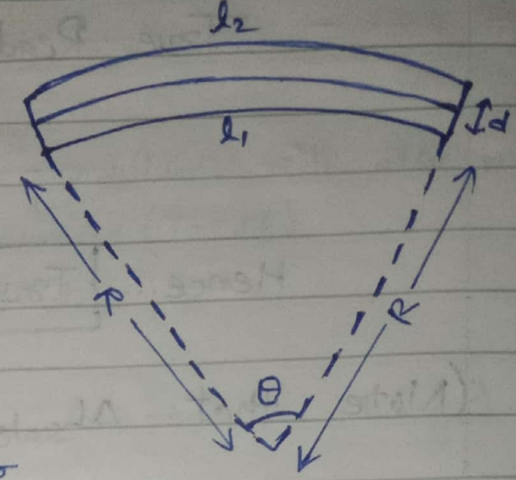


As temp. ↓es above 4°c...
Water contracts and become heavy and goes down.
⇒ Phir niche wala upar aake contract ↓

एक time aayega जब saara paani 4°c पर आजाएगा।
Surface wallah 4°c से नीचे expand करेगा और उपर ही रहेगा.
⇒ Temp. 0°c होते ही surface wallah paani ice ban jaayega.
⇒ Surface के niche paani most 4°c पे
⇒ मछली Zinda hai.

BIMETALIC STRIP :-

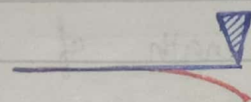




→ If $\alpha_2 > \alpha_1$ then the 2nd rod will expand more and $l_2 > l_1$
Hence l_2 को बड़ा रखने के लिए मुड़ेगी ऐसे... जैसे shown in figure.

$$\left(\begin{aligned} l_2 &= (R+d)\theta \\ l_1 &= R\theta < l_2 \end{aligned} \right)$$

→ Used in fire alarm.



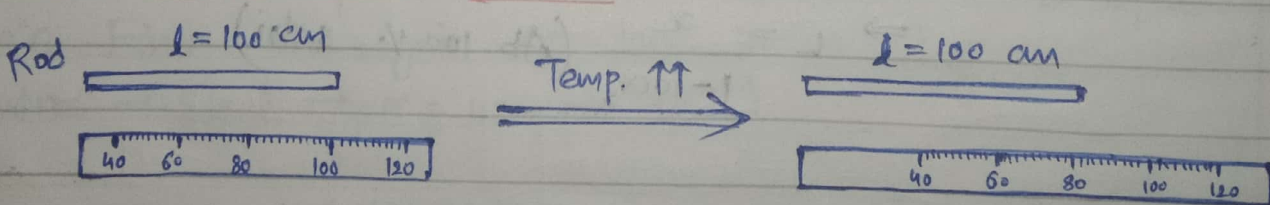
Temp. ↑↑ ⇒ Strip bent
⇒ Loose contact
⇒ Alarm sounds

$$\frac{l_2}{l_1} = \frac{(R+d)\theta}{R\theta} = \frac{L\alpha_2\Delta T}{L\alpha_1\Delta T} \Rightarrow \frac{R+d}{R} = \frac{\alpha_2}{\alpha_1}$$

$$\Rightarrow \alpha_1 R + \alpha_1 d = \alpha_2 R$$

$$\Rightarrow \boxed{R = \frac{d\alpha_1}{\alpha_2 - \alpha_1}}$$

FAULT IN METALLIC SCALES :-



Reading = 80 cm

→ Assuming length of rod doesent change... Scale measures shorter length of the rod than actual.

True Reading = 100 = $\underset{\text{Measured length}}{80}$ + $\underset{\text{Rise in length of scale}}{20}$

Hence... $\boxed{\text{True reading} = M.L. + \Delta L_{scale}}$

(Note that ΔL_{scale} will be −ve if temp. is lowered.)

Q. A steel scale calibrated for 20°C is used to measure the length of a rod at 30°C which comes out to be x. Find the true length of rod if α_s is given for scale.
(Length of scale = 100 cm)
⇒ Koi fark नहीं पड़ता

Sol. ~~True reading = M.L + ΔL_{scale}~~

$\Rightarrow$ ~~$L = x + L\alpha_s(30-20)$~~

$\Rightarrow$ ~~$L = x + 10L\alpha_s$~~

$\Rightarrow$ ~~$L(1-10\alpha_s) = x$~~

$\Rightarrow$ ~~$L = \dfrac{x}{1-10\alpha_s}$~~

$\Rightarrow L = x + 100\alpha_s(30-20)$

$\Rightarrow L = x + 1000\alpha_s$

~~$\Rightarrow L = \dfrac{x}{1-1000\alpha_s}$~~ Phir Se Galti !!

$\boxed{L = x + 1000\alpha_s}$

(000 % correct उत्तर)

L ✗ Length of scale

* → Steel Scale ke utne hi part ka expansion karo ki jitne ki actual rod ho.

⇒ Pehle wallah hi sahi tha.

$\Rightarrow L = \dfrac{x}{1-10\alpha_s}$ (Ab 100 % sahi)

NOTE: 1) Temp ↑↑ ⇒ Readings streched ⇒ Rod Gota
2) Temp ↓↓ ⇒ Readings compressed ⇒ Rod Lamba
3) For same rise/fall... kitna lamba/Gota = $l\alpha\Delta T$
(Rod ka actual length)

PENDULUM BASED CLOCKS :-

→ If temp. is increased length of pendulum will also increase and hence time period will also increase ($T \propto \sqrt{l}$).

⇒ Slow down. (Late)

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{L(1+\alpha\Delta\theta)}{g}}$$ ($\Delta\theta \rightarrow$ temp. change)

Hence... $\frac{T'}{T} = \sqrt{1+\alpha\Delta\theta} = (1+\alpha\Delta\theta)^{1/2}$

As α is very small... $\frac{T'}{T} = 1 + \frac{\alpha}{2}\Delta\theta$

$$\Rightarrow \boxed{T' = T + \frac{T}{2}\alpha\Delta\theta}$$

Put $\Delta\theta$ −ve if temp. is lowered.

→ Time lost/gain in measuring one time period...

$$(\Delta T) = \frac{1}{2}T\alpha\Delta\theta$$

$$\Rightarrow \boxed{\text{Time lost/gain during 't' sec} = \frac{t}{2}\alpha\Delta\theta}$$

+ve (Lost)
−ve (Gain)

Q. For a second's pendulum calibrated at 20°C is operated at 30°C. Find the gain or loss in 1 day.
(Wire → Steel → $\alpha = 1.2 \times 10^{-5}\ °C^{-1}$)

Sol.

$$\Delta t = \frac{t}{2}\alpha\Delta\theta = \frac{1 \times 24 \times 3600}{2} \times 1.2 \times 10^{-5} \times \underbrace{(30-20)}_{+ve \Rightarrow \text{Lost}}$$

$$= 5.184 \text{ sec.}$$

EFFECT ON DENSITY OF LIQUID :-

$$(m, V, \gamma) \xrightarrow{\Delta T} (m, V+\Delta V, \gamma)$$

$$\rho = \frac{m}{V} \qquad \rho' = \frac{m}{V+\Delta V}$$

Now... $\frac{\rho'}{\rho} = \frac{V}{V+\Delta V} = \frac{V}{V+V\gamma\Delta T} = (1+\gamma\Delta T)^{-1}$

$$\Rightarrow \rho' = \rho(1+\gamma\Delta T)^{-1}$$

$$\Rightarrow \boxed{\rho' = \rho(1-\gamma\Delta T)}$$ (as γ is very small)

(Put ΔT with signs)

SPECIFIC HEAT CAPACITY :- (Gota 'c')

→ It is the amount of heat required to raise the temp. of 1 g substance by 1°C.

e.g. $c_{water} = 4.2\ J/g°C$ (High) ⇒ Do not heat/cool rapidly

$c_{copper} = 0.4\ J/g°C$ (low) ⇒ Heats and cools rapidly

→ Higher SHC means more heat absorbs means less heat conducts and hence bad conductor.

High HSC ⇒ Bad conductor

Low HSC ⇒ Good conductor

$c = \frac{Q}{m\Delta T}$ ⇒ $\boxed{Q = mc\Delta T}$

mass

|Change in temp.|

SHC

Heat absorbed/Released

(Use when temp. is changing)

Per unit mass and per unit temp. rise/fall

Q. Find the amount of heat required to heat 2g water from 10°c to 40°C.

Sol: $Q = mc\Delta T = 2 \times 4.2 \times 30 = 252\ J$

Q. Find the amount of heat released when 2 g water is cooled from 70°C to 10°C.

Sol. $Q = mc\Delta T = 2 \times 4.2 \times 60 = 506\ J$

Q. A heater gives heat at rate of 100 W. It is used to heat 50 g water from 20°C to 30°C. Find the time taken to do this.

Sol. Heat given by heater = Heat absorbed by water.

$\Rightarrow 100t = mc\Delta T$

$\Rightarrow 100t = 50 \times 4.2 \times 10$

$\Rightarrow t = 21\ sec.$

Q. A refregerator extracts energy at rate of 100 W. How much time will it take to cool a lemon squash from 30°C to 5°C. ($C_{lemon} = 4.2\ g^{-1}\,°C^{-1}.J$) (Mass = 100 g)

Sol. $100t = mc\Delta T \Rightarrow 100t = 100 \times 4.2 \times 25 \Rightarrow t = 105\ sec.$

PRINCIPLE OF CALORIMETRY / METHOD OF MIXTURES :-

$$\boxed{\text{Heat loss by Hot body} = \text{Heat gained by Cold body}}$$

(Based on law of conservation of energy)

Q. 10 g water at 80°C is mixed with 50 g sugar at 10°C. Find the temp. of mixture. ($C_{water} = 4.2$ J/g°C, $C_{copper} = 0.4$ J/g°C)

Sol. Heat lost by water = Heat gain by copper

$$\Rightarrow 10 \times 4.2 \underbrace{(80-\theta)}_{\text{Always +ve}} = 50 \times 0.4 \underbrace{(\theta - 10)}_{\text{Always +ve}}$$

$\Rightarrow \theta =$

Q. 20 g of solid at 20°C is mixed with 100 g of cold water at 5°C. If final temp. of mixture is 10°C, Find SHC of solid.

Sol. Heat loss by solid = Heat gained by water

$\Rightarrow 20 \times C \times (20-10) = 100 \times 4.2 \times (10-5)$

$\Rightarrow C =$

Q. In a copper vessel; we pour 20 g of water at 80°C. Now 100 g of cold water at 10°C is added to vessel. Find the final temp. of mixture if vessel weighs 50 g. ($C_{copper} = 0.4$ J/g°C)

Sol. Temp. of vessel = Temp. of content.

$\Rightarrow$ Hot bodies are both hot water and copper vessel.

$\Rightarrow$ Heat lost by hot water + Heat lost by copper

= Heat gained by cold water.

$\Rightarrow 20 \times 4.2(80-\theta) + 50 \times 0.4(80-\theta) = 100 \times 4.2(\theta - 10)$

$\Rightarrow \theta =$

Q. The temp. of 600 g of cold water rises by 15°C when 300 g of hot water is added. Find initial temp. of cold water. (Temp. of hot water = 50°C) sol.

$Q_{\text{hot water}} = Q_{\text{cold water}}$

$\Rightarrow 300 \times c \times (50 - (x+15)) = 600 \times c \times (x + 15 - x)$

$\Rightarrow x = 5°C.$

($x \rightarrow$ Initial temp. of cold water
$(x+15) \rightarrow$ Eq^m temp.)

HEAT CAPACITY :- (Thermal Capacity)

→ Amount of heat required to raise the temp. of whole body by 1°C. (Bada C)

$$\boxed{C = mc}$$

e.g. Heat capacity of 20 g water $= 20 \times 4.2 = 84$ J

→ SHC does not depend on mass but heat capacity depends on the given mass of substance.

→ As... $Q = mc\Delta T \Rightarrow \boxed{Q = C\Delta T}$

→ Bada C

LATENT HEAT :-

↳ Gupi Hui (Jo dikhai na de)

→ Latent heat is a hidden heat used when a substance changes its state.

→ When we give heat during phase change the temp. doesn't change. This heat is used to break/weaken the bonds for changing the phase.

$$\text{Ice } (0°C) \xrightarrow[\text{Heat}]{\text{Latent}} \text{Water } (0°C)$$

⇒ Specific latent heat :- To change the phase of 1 g substance.

⇒ Specific latent heat of fusion of ice :-

$$\underset{(1\,g)}{\text{Ice}} \xrightarrow[336\,J]{\text{Heat}} \underset{(1\,g)}{\text{Water}}\ (0^\circ C)$$

$$\boxed{L_f = 336\ J/g = 80\ cal/g}$$

⟶ Melting

Fusion ⟵

⇒ Specific latent heat of Vapourisation of water :-

$$\underset{(1\,g)}{\text{Water}} \xrightarrow{\text{Heat}} \underset{(1\,g)}{\text{Water vapours}}\ (100^\circ C)$$

⟶ Vapourisation / Boiling

Condensation ⟵

$$\boxed{L_v = 2268\ J/g = 540\ cal/g}$$

→ As spesific latent heat is the heat required per unit mass...

$$L = \frac{Q}{M} \Rightarrow \boxed{Q = mL}$$

Q → Heat, m → Mass, L → Specific latent heat (Use for phase change)

Q. Find the amount of heat required to change 10 g of ice at $-5^\circ C$ to water at $20^\circ C$. ($SHC_{water} = 4.2\ J/g^\circ C$) ($SHC_{ice} = 2.1\ g^{-1}\,{}^\circ C^{-1}\cdot J$) ($SLH_{fusion} = 336\ J/g$).

Sol.

$$\underset{\substack{(-5^\circ C)\\ 10g}}{\text{Ice}} \xrightarrow[Q_1]{\text{Heat}} \underset{\substack{(0^\circ C)\\ 10g}}{\text{Ice}} \xrightarrow[Q_2]{\text{Heat}} \underset{\substack{(0^\circ C)\\ 10g}}{\text{Water}} \xrightarrow[Q_3]{\text{Heat}} \underset{\substack{(20^\circ C)\\ 10g}}{\text{Water}}$$

Total heat required

$= Q_1 + Q_2 + Q_3$

$= (10)(2.1)(5) + 10(336) + 10(4.2)(20)$

$=$

Q. Find the amount of heat released when 1 kg of water vapour is cooled to water at 20°C.
$(SHC_{water} = 4.2\ J/g°C)$ $(SLH_{vap} = 2260\ J/g)$.

Sol.

$$\underset{\substack{(100°C)\\ 1\,kg}}{\text{Water Vapour}} \xrightarrow{Q_1} \underset{\substack{(100°C)\\ 1\,kg}}{\text{Water}} \xrightarrow{Q_2} \underset{\substack{(20°C)\\ 1\,kg}}{\text{Water}}$$

Total heat released

$= Q_1 + Q_2$

$= (1000 \times 2260) + (1000)(4.2)(80)$

$=$

Q. A piece of ice of mass 40 g is added to 200 g of water at 50°C. Calculate the final temp. when all ice is melted.
$(SHC_{water} = 4.2\ J/g \cdot K)$ $(SLH_{ice} = 336\ J/g)$.

Sol.

$$\underset{(200\,g)}{\overset{(50°C)}{\text{Water}}} \xrightarrow{mC\Delta T} \theta \xleftarrow{mC\Delta T} \underset{(0°C)}{\overset{(40\,g)}{\text{Water}}} \xleftarrow{mL} \underset{(40\,g)}{\overset{(0°C)}{\text{Ice}}}$$

Heat released by water = Heat gained by ice

$\Rightarrow mC\Delta T = mL + mC\Delta T$

$\Rightarrow (200)(4.2)(50-\theta) = (40)(336) + (40)(4.2)(\theta - 0)$

$\Rightarrow \theta =$

Q. Result of mixing 10 g ice at −10°C with 10 g water at 10°C.
$(C_w = 4.2\ J/g°C,\ C_i = 2.1\ J/g°C,\ L_f = 336\ J/g)$.

Sol (10 g) Water (10°C) $\longrightarrow$ Water (θ) $\longleftarrow$ Water (0°C) $\longleftarrow$ Ice (0°C) $\longleftarrow$ Ice (−10°C) (10 g)

$\Rightarrow 10 \times 4.2 \times (10-\theta) = 10 \times 4.2 \times \theta + 10 \times 336 + 10 \times 2.1 \times 10$

$\Rightarrow 42(10-\theta) = 42\theta + 3360 + 210$

$\Rightarrow \theta = -ve$ ✗ (Kuch to gadbad hai... Daya !!)

आओ देखें !

Max. heat released by water when it come to 0°C

$= mc\Delta T = 10 \times 4.2 \times 10 = 420$ J

Heat required by ice to reach zero

$= mc\Delta T = 10 \times 2.1 \times 10 = 210$ J

Heat remaining $= 420 - 210 = 210$ J

If all ice melts then heat required $= mL = 10 \times 336 = 3360$ J

$\Rightarrow$ All ice will not melt

Max. mass of ice that can be melted $(m) = \frac{210}{L} = \frac{210}{336} = 0.625$ g

$\Rightarrow$ Some ice is still remaining. $\Rightarrow T_f = \theta = 0°C$.

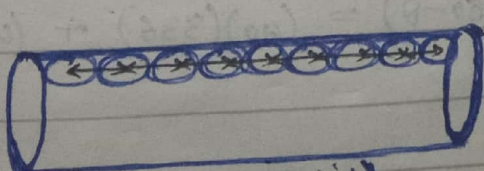
CONDUCTION :- (Generally in solids)

→ It is a way of heat transfer by molecular collisons.

→ No actual flow of matter.

$(T_1 > T_2)$

T_1



T_2

(Metal Solid)

→ Molecules vibrates and transfers energy to neighbouring molecules by just vibrating collisons.

⇒ **Variable State :-**

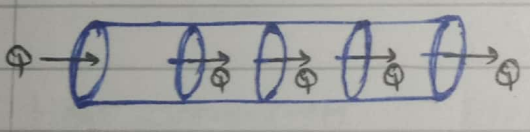
→ Heating rod from one end... har cross-section में molecul kuch heat absorb karke temp. बढ़ाएँगे aur kuch heat pass kar denge.

→ Temp. of each cross-section samy ke sath badalta hai.

⇒ **Steady State :-** ~~Temp. of each point will be same.~~

→ Temp of each cross-section remains constant w.r.t time.

→ Koi bhi cross-section heat absorb नहीं करेगा।

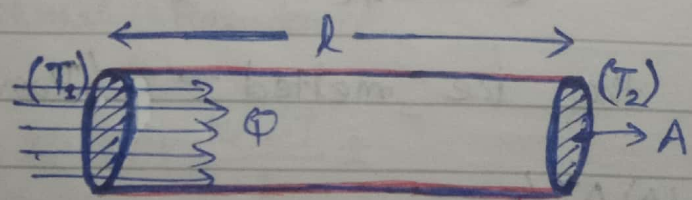


(Heat passing through each cross-section is same.)

→ Steady state me heat flow hoti hai.

→ During steady state... $\left(\frac{dT}{dx}\right)_t \neq 0$ but $\left(\frac{dT}{dt}\right)_x = 0$.

⇒ **Maths :-**



$(Q \perp A)$

Insulating material (So that the heat does not lose by radiation)

Q → Amount of heat that flows in time 't'.

→ It is experimentally observed that...

$$\left.\begin{aligned} Q &\propto A \\ Q &\propto (T_1 - T_2) \\ Q &\propto t \\ Q &\propto \frac{1}{l} \end{aligned}\right\} \quad \boxed{Q = k\,\frac{A(T_1 - T_2)}{l}\,t}$$

k → Thermal conductivity (Constant for a material)

SI unit: $\frac{J}{m\,s\,°C}$ (or) $\frac{W}{m\,°C}$

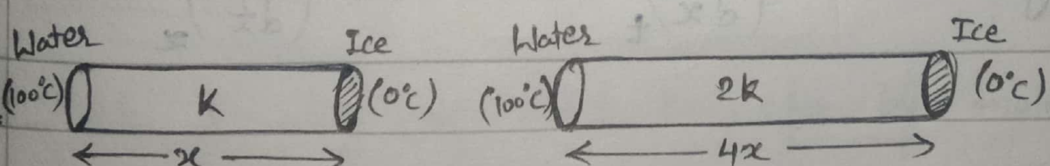
Common unit: $\frac{cal}{cm \cdot s \cdot °C}$

⇒ **Heat Current (H)** :- Rate of flow of heat (Heat per second)

$$H = \frac{Q}{t} \Rightarrow \boxed{H = \frac{KA\Delta T}{l}} \qquad \left(\text{Differential form} \rightarrow H = \frac{dQ}{dt}\right)$$

Q. A copper cube ($a = 10$ cm)
Temp. of two ends ($100°C$, $0°C$)
$K_{copper} = 400\ W/m°C$
What is the rate of heat flow?

Sol. $H = \frac{KA\Delta T}{l} = \frac{400 \times (0.1)^2 \times 100}{0.1} = 4000\ W$

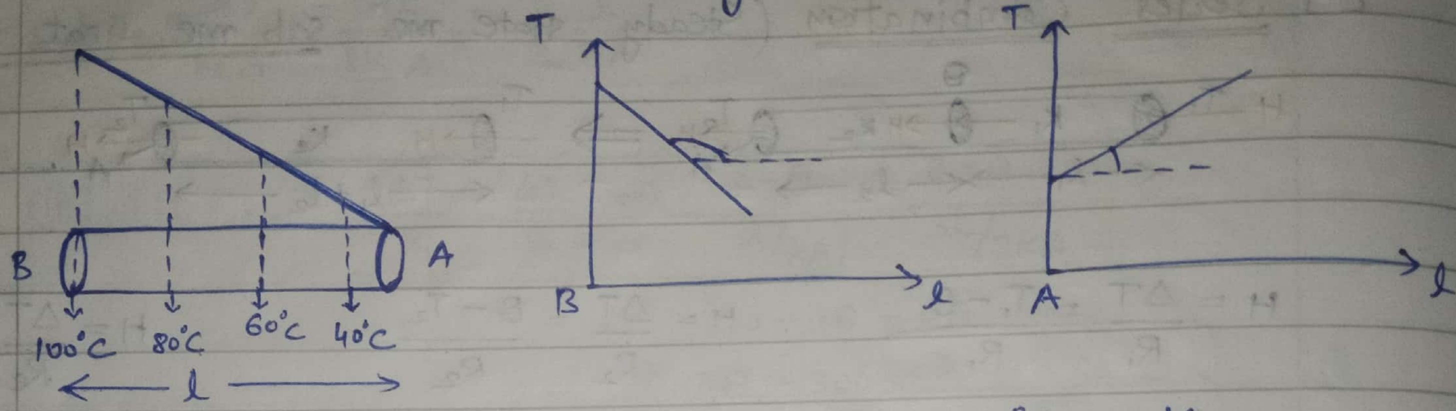
Q.


$\frac{m_1}{m_2}$ → Ratio of the ice melted → ? (Provided 't' same)

Sol. $\frac{Q_1}{Q_2} = \frac{H_1 t}{H_2 t} = \frac{k(A)\Delta T/x}{2k(A)\Delta T/4x} = 2$

$\Rightarrow \frac{m_1 L_f}{m_2 L_f} = 2 \Rightarrow \frac{m_1}{m_2} = 2$

⇒ **Temperature gradient** :- It is assumed that temp. varies linearly with length at steady state.



$$\text{Temp. gradient} = \frac{dT}{dl} = \frac{°C}{m} \text{ or } \frac{K}{m}$$

⇓ Constant as per assumption

(slope)

⇒ **Analogy** :- (बहुत ही प्यारा analogy)

Heat	Electricity
Heat current → H	Electrical current → i
Why ? → ΔT	Why ? → ΔV
$H = \frac{KA\Delta T}{l} = \frac{\Delta T}{(l/KA)}$	$i = \frac{\Delta V}{R_e}$
$\boxed{R_{th} = \frac{l}{KA}}$	$R_e = \frac{\rho l}{A}$

R_{th} → Thermal Resistance

$$\boxed{H = \frac{\Delta T}{R_{th}}}$$ Ratt lo

⇒ **Summary** :-

1) $\Phi = \frac{KA\Delta T}{l} t$

2) $H = \frac{\Phi}{t} = \frac{KA\Delta T}{l} = \frac{d\Phi}{dt}$

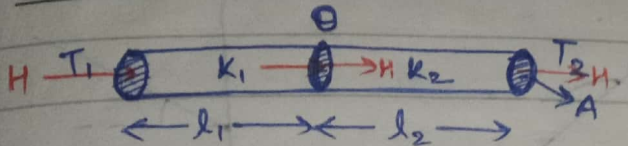
3) Temp. gradient $= \frac{dT}{dl}$

4) $R_{th} = \frac{l}{KA}$

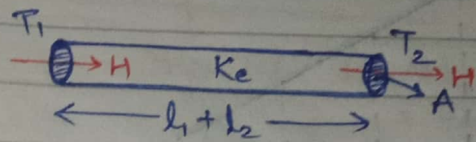
5) $\frac{\Delta T}{V} = \frac{H R_{th}}{i R}$

COMBINATION OF CONDUCTORS :-

[1] Series Combination (Steady state me sab me heat same)



$\Rightarrow$



$$H = \frac{\Delta T}{R_1} = \frac{T_1-\theta}{R_1} \qquad H = \frac{\Delta T}{R_2} = \frac{\theta - T_2}{R_2} \qquad H = \frac{\Delta T}{R_e} = \frac{T_1-T_2}{R_e}$$

$$\Rightarrow T_1 - \theta = HR_1 \quad \text{—(1)} \qquad \Rightarrow \theta - T_2 = HR_2 \quad \text{—(2)} \qquad \Rightarrow T_1 - T_2 = HR_e \quad \text{—(3)}$$

(1) + (2) → (3) ... $T_1 - \theta + \theta - T_2 = HR_1 + HR_2$

$$\Rightarrow T_1 - T_2 = HR_e = H(R_1+R_2)$$

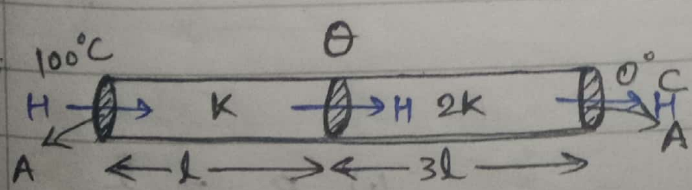
$$\Rightarrow \boxed{R_e = R_1 + R_2}$$

As... $R = \dfrac{l}{KA} \Rightarrow \dfrac{l_1+l_2}{K_eA} = \dfrac{l_1}{K_1A} + \dfrac{l_2}{K_2A}$

$$\Rightarrow \frac{l_1+l_2}{K_e} = \frac{l_1}{K_1} + \frac{l_2}{K_2}$$

$$\Rightarrow \boxed{K_e = \frac{l_1+l_2}{\dfrac{l_1}{K_1}+\dfrac{l_2}{K_2}}}$$

Q.



Find (i) R_e
(ii) K_e
(iii) θ

Sol. (ii) $R_e = R_1 + R_2 \Rightarrow \dfrac{4l}{K_eA} = \dfrac{l}{K_1A} + \dfrac{3l}{2KA}$

$$\Rightarrow \frac{4}{K_e} = \frac{1}{K} + \frac{3}{2K} \Rightarrow K_e = \frac{8K}{5}$$

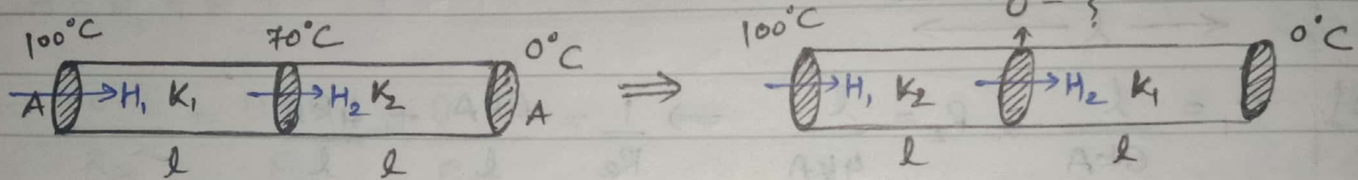
(ii) $$R_e = \frac{4l}{K_e A} = \frac{4l}{\frac{8K}{5} A} = \frac{5}{2}\frac{l}{KA}$$

(iii) $$H_1 = H_2 \Rightarrow \frac{\Delta T}{R_1} = \frac{\Delta T}{R_2} \Rightarrow \frac{100-\theta}{l/KA} = \frac{\theta - 0}{3l/2KA}$$

$$\Rightarrow 300 - 3\theta = 2\theta$$

$$\Rightarrow 5\theta = 300 \Rightarrow \theta = 60\,^\circ C$$

Q.



Sol.

$$H_1 = H_2$$

$$\Rightarrow \frac{\Delta T}{R_1} = \frac{\Delta T}{R_2}$$

$$\Rightarrow \frac{100-70}{\frac{l}{K_1 A}} = \frac{70-0}{\frac{l}{K_2 A}}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{7}{3}$$

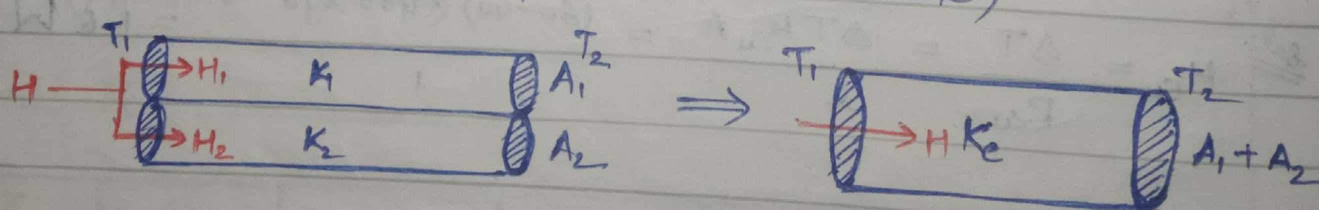
$$H_1 = H_2$$

$$\Rightarrow \frac{100-\theta}{\frac{l}{K_2 A}} = \frac{\theta - 0}{\frac{l}{K_1 A}}$$

$$\Rightarrow 100 - \theta = \theta \times \frac{7}{3}$$

$$\Rightarrow 10\theta = 300 \Rightarrow \theta = 30^\circ C$$

2] Parallel Combination (Temp. difference same)



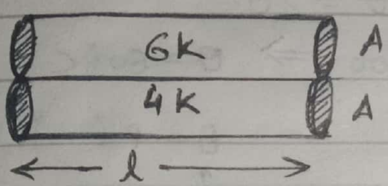
$$H = \frac{T_1 - T_2}{R_e} \; ; \; H_1 = \frac{T_1 - T_2}{R_1} \text{ and } H_2 = \frac{T_1 - T_2}{R_2}$$

Also... $H = H_1 + H_2$

$$\frac{T_1 - T_2}{R_e} = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} \Rightarrow \boxed{\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}}$$

As $R = \frac{l}{KA}$ ... $\frac{K_e(A_1 + A_2)}{l} = \frac{K_1 A_1}{l} + \frac{K_2 A_2}{l}$

$$\Rightarrow \boxed{K_e = \frac{K_1 A_1 + K_2 A_2}{K_1 + K_2}}$$

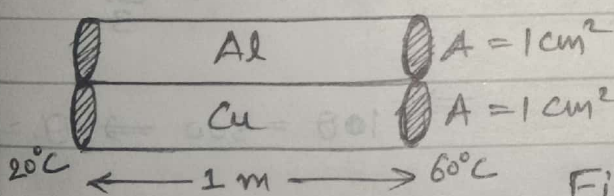
Q. 

$R_e = ?$; $K_e = ?$

Sol. $R_1 = \frac{l}{6KA}$; $R_2 = \frac{l}{4KA}$ $\Rightarrow \frac{1}{R_e} = \frac{6KA}{l} + \frac{4KA}{l}$

$$\Rightarrow \frac{K_e(2A)}{l} = \frac{6KA}{l} + \frac{4KA}{l}$$

$$\Rightarrow K_{eq} = 5K \text{ and } R_e = \frac{l}{K_e(2A)} = \frac{l}{5K(2A)} = \frac{l}{10KA}$$

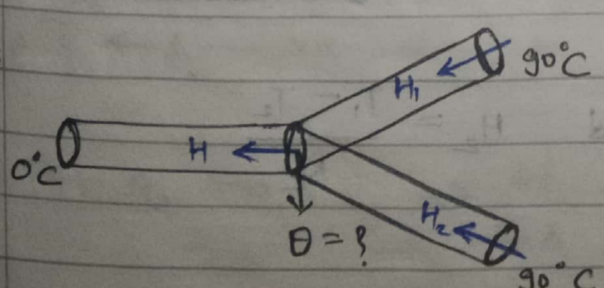
Q. 

$K_{Al} = 200$ W/m°C

$K_{Cu} = 400$ W/m°C

Find the rate of heat flow through copper rod.

Sol. $H_{Cu} = \frac{\Delta T}{R_{Cu}} = \frac{\Delta T K_{Cu} A}{l} = \frac{(60-40) \times 400 \times 10^{-4}}{1} = 1.6$ W

Q. 

Sol. $H_1 + H_2 = H$

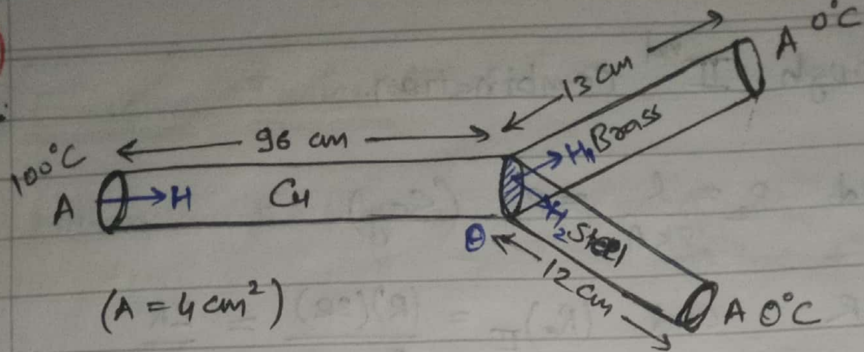
$$\Rightarrow \frac{90 - \theta}{R} + \frac{90 - \theta}{R} = \frac{\theta - 0}{R}$$

$$\Rightarrow 180 - 2\theta = \theta$$

$K, l, A \longrightarrow$ Same. $\Rightarrow R$ same $\Rightarrow \theta = 60°C$

(MAINS 2014)

Q.



$(A = 4\ cm^2)$

$K_{cu} = 0.192$ cal/cm °C·s

$K_{brass} = 0.26$ cal/cm °C·s

$K_{stell} = 0.12$ cal/cm °C·s

Find the rate of heat flow through copper rod in cal/s. ?

sol. $H = H_1 + H_2$

$$\Rightarrow \frac{100-\theta}{R_{cu}} = \frac{\theta - 0}{R_{Br}} + \frac{\theta - 0}{R_{st}}$$

$$\Rightarrow \frac{(100-\theta)0.192A}{96} = \frac{\theta \cdot 0.26A}{13} + \frac{\theta \cdot 0.12A}{12}$$

$$\Rightarrow 200 - 2\theta = 2\theta + \theta$$

$$\Rightarrow \theta = 40°C$$

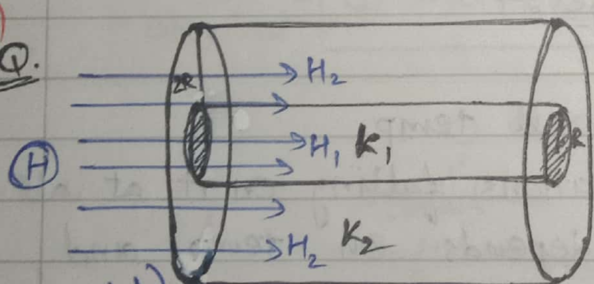
Hence... $H_{cu} = \frac{\Delta T_{cu}}{R_{cu}}$

$= \frac{100-40}{(96)/(0.192)4}$

$= 4.8$ cal/s

(JEE 2001)

Q.



(Parallel) (Annular cylinder)

Find K_e.

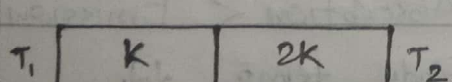
sol. $K_e = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

$= \frac{K_1(\pi R^2) + K_2\pi((2R)^2 - R^2)}{\pi R^2 + \pi((2R)^2 - R^2)}$

$= \frac{K_1 + 3K_2}{4}$

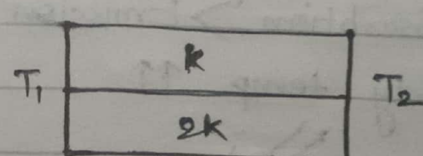
(ADV. 2013)

Q.



(I)

(l, A same)



(II)

In 9 sec, certain amount of heat flows through I^{st} Combination. Find the time in which same amount of heat flows through II^{nd} combination.

sol. $R_1 = \frac{l}{KA} = 2R$ and $R_2 = \frac{l}{2KA} = R$ (Say)

$(R_e)_I = R_1 + R_2 = 3R$ and $(R_e)_{II} = \frac{(R)(2R)}{R+2R} = \frac{2R}{3}$

$Q = Ht$
$= \frac{\Delta T}{3R} \times 9$

$Q = Ht$
$= \frac{\Delta T}{2R/3} \times t$

As Q is same... $\frac{\Delta T \times 3}{R} = \frac{3\Delta T}{2R} \times t \Rightarrow t = 2$ sec.

RADIATION :-

→ It is a way of heat transfer which requires no medium as it is an electromagnetic waves.

⇒ Prevost's theory of exchange :-

1.) All body emits radiation at all temp.
2.) All body absorbs heat radiations falling on it at all temp.
3.) Amount of emission/absorbtion depends on temp. and material of body.

$T_{surr} > T_{body}$
⇒ Absorbtion > Emission
⇒ Body temp. ↑↑

$T_{surr} < T_{body}$
⇒ Absorbtion < Emission
⇒ Body temp. ↓↓

Body temp. Const.
⇒ $T_{surr} = T_{body}$
⇒ Emission = Absorbtion

⇒ **Emissive Power (E) :-**

Heat emitted by a body... $\left.\begin{array}{l}\phi \propto A \\ \phi \propto t\end{array}\right\}$ $\boxed{\phi = EAt}$

E → Emissive Power → Unit: $\frac{J}{s\,m^2}$ or $\frac{W}{m^2}$

Depends on
(i) Material
(ii) Wavelength
(iii) Temp. (Directly)

$\boxed{E_\lambda}$ → Spectral emissive power
→ Alag alag wavelength ke liye alag alag emit karne ki tendency.

⇒ **Absorbtive power (a) :-** %age of heat absorbed out of total heat incident.

$$a = \frac{\text{Heat absorbed by body}}{\text{Heat incident on body}} \quad \text{(Unitless)}$$

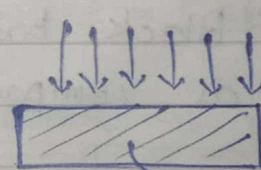
Heat absorbed ← $\boxed{\phi = a\,(\text{Heat incident})}$

a → (i) Material
(ii) Wavelength

$$0 \leq a \leq 1$$

0 → Ideal Reflector
1 → Perfect reflector

100 Radiations



60 radiations absorbed

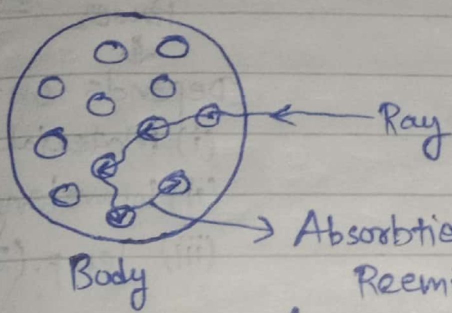
$$\Rightarrow a = \frac{60}{100} = 0.6 \text{ or } 60\%$$

$\boxed{a_\lambda}$ → Spectral absorbtive power

NOTE: Substance jis colour ka hoga uss wavelength ke liye emissive power jyada aur baki sab ke liye absorbtive power jyada.

BLACK BODY :- (Sun/Stars)

Actual → An ideal black body absorbs all radiations falling on it completely at any temp and at any incident angle.



→ Absorbtion and Reemission (But never reflected)

i) $a_\lambda = 1$
ii) Zero reflection ✓
iii) Zero transmittence
iv) Zero emittance ✗

→ Black color ki body → Can be black body

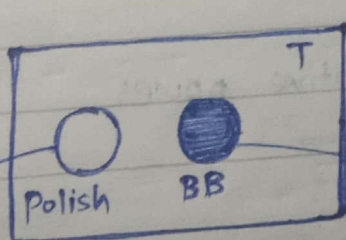
But.... black body → Need not be necessarily black

No reflection → Black → Dusre ke colour ko wapas nahi bhejta.

But emission → Temp. ki wajah se khud ki waves de sokta hai.

Depends on temp. ⇒ Red/Blue/Yellow anything.

Secondary → Black body emits maximum radiations at a given temp. as compared to other bodies.



Both are in thermal eqm.
⇒ Jitni absorb utni emit

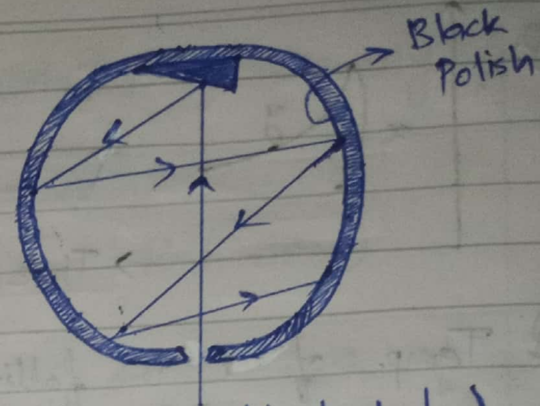
Absorbs more $(a=1)$
⇓
Will emit more

Absorbs less $(a<1)$
⇓
Will emit less

Secondary definition explained.

⇒ Practical black bodies :-

1) Lamp black (1% reflection)
2) Graphite (3% reflection)
3) Platinum / Gold black
4) An Isothermal enclosure ⇒



(Nearly black body)
⇒ No reflection

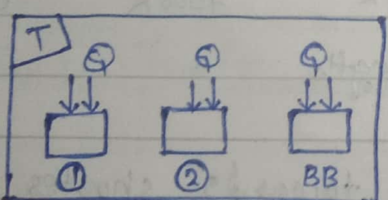
KIRCHOFF'S LAW :-

→ Good absorbers are good emitters.

Absorber ↑↑ ⇒ $a \uparrow$
Emitter ↑↑ ⇒ $E \uparrow$ } For any body... $\boxed{E \propto a}$

→ Also... $\boxed{E_\lambda \propto a_\lambda}$ "Lal rang jitna absorb karega temp. badhne par utna hi emit karega".

Proof:



a_1 a_2 $a = 1$

E_1 E_2 E_b

All have same dimensions.
⇒ Q → same

Heat absorbed = a (Heat incident)
Heat emitted = EAt

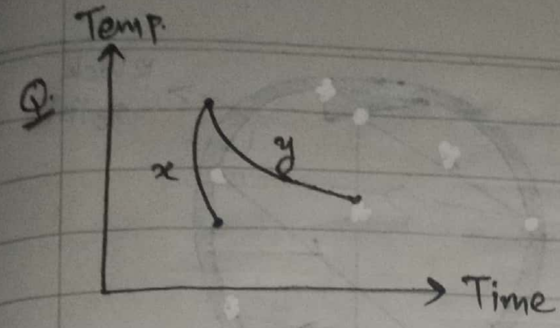
Thermal eqm
⇓
$Q_{absorbed} = Q_{emitted}$

$a_1 Q = E_1 A t$ $a_2 Q = E_2 A t$ $1 \times Q = E_b A t$

$\Rightarrow \frac{a_1}{E_1} = \frac{At}{Q}$ $\Rightarrow \frac{a_2}{E_2} = \frac{At}{Q}$ $\Rightarrow \frac{1}{E_b} = \frac{At}{Q}$

Hence... $\frac{E_1}{a_1} = \frac{E_2}{a_2} = E_b \Rightarrow \boxed{\frac{E_\lambda}{a_\lambda} = E_{\lambda b}} \Rightarrow \frac{E_\lambda}{a_\lambda} = \text{Constant} \Rightarrow E_\lambda \propto$

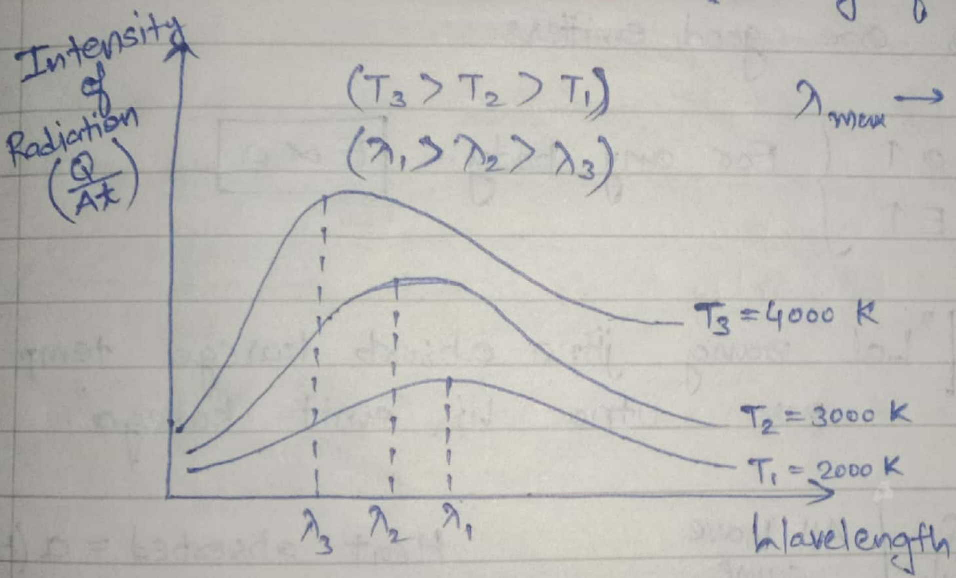
(At a given temp.)

Q. 

Give relation b/w E_x, E_y and a_x, a_y in inequality.

Sol. Temp. of x is falling rapidly.
$\Rightarrow$ Emits heat faster
$\Rightarrow E_x > E_y$
$\Rightarrow a_x > a_y \quad (\because E \propto a)$

WIEN'S DISPLACEMENT LAW :- (Ideally followed by black body)



$\lambda_{max} \rightarrow$ No. of radiation of this λ is maximum.

Temp.	λ_{max}
2000 K	1.5×10^{-6} m
3000 K	1×10^{-6} m
4000 K	0.75×10^{-6} m

→ As temp. increases, λ_{max} displaces towards shorter wavelength

→ Here we can see that.. $\lambda_1 T_1 = \lambda_2 T_2 = \lambda_3 T_3$

High Wavelength
⇓
Low temp.
⇓
Low energy

$\Rightarrow \lambda_{max} \cdot T = \text{constant} = b \rightarrow$ Wein's constant

$\Rightarrow \boxed{T = \frac{b}{\lambda_{max}}}$

($b = 2.898 \times 10^{-3}$ m-K for black body

→ Used to measure temp. of distant stars.

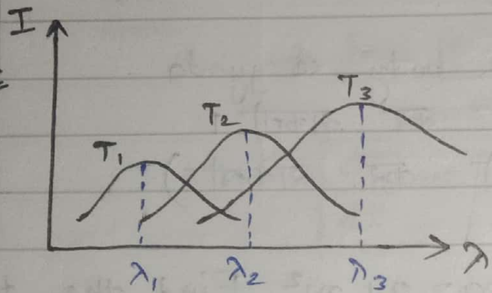
→ Area under graph gives total emission of intensity from body at that temperature.

$\Rightarrow T_3 > T_2 > T_1 \Rightarrow I_3 > I_2 > I_1$

(Higher temp. → More Radiations)

→ In all cases... it is not necessary that peak of graph goes upward with rise in temp. but area will surely increase.

(JEE 2002) Q.



sol. $\lambda_{max} \rightarrow \lambda_3 > \lambda_2 > \lambda_1$

$\Rightarrow T_3 < T_2 < T_1$

(By wein's law)

(IIT 1998) Q.

Sun

$\lambda_{max} = 510 nm$

North Star

$\lambda_{max} = 350$ nm

Find $T_{sun}/T_{north\ star}$.

sol. $\dfrac{T_{sun}}{T_{north\ star}} = \dfrac{\lambda_{north\ star}}{\lambda_{sun}}$ $\left(\because \lambda \propto \dfrac{1}{T}\right)$

$\Rightarrow \dfrac{T_{sun}}{T_{north\ star}} = \dfrac{350}{500} = 0.69$

STEFAN'S - BOLTZMAN LAW :-

→ For a perfectly black body...

Intensity of radiation $\propto T^4$

$\Rightarrow \dfrac{Q}{A \times t} \propto T^4$

$\Rightarrow \boxed{\dfrac{Q}{At} = \sigma T^4}$

(σ → Universal Constant
→ 5.67×10^{-8} W/m^2K^4
→ 6×10^{-8} W/m^2K^4)

$$\Rightarrow \frac{Q}{t} = \sigma A T^4$$

$$\Rightarrow \boxed{\text{Power Radiated} = \sigma A T^4}$$ (For perfectly black body)

→ For any other body... $\boxed{P = e\sigma A T^4}$

e → emissivity (For a given material)

$(0 \leq e \leq 1)$

⇓

Koi bhi body black body se jyada heat emit नहीं कर सकती ।
(∵ Absorb नहीं कर सकती)

Q. Heater : $P_{radiated} = 6000\ W$, coil area $= 0.1\ m^2$, Find the temp. of coil assuming it as a black body.

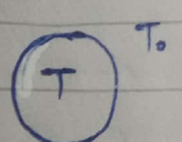
$(\sigma = 6 \times 10^{-8}\ W/m^2 \cdot K^4)$

Sol.

$$P = \sigma A T^4 \Rightarrow 6000 = 6 \times 10^{-8} \times 0.1 \times T^4$$

$$\Rightarrow T = 1000\ K$$

⇒ **When surrounding temp. is given :-**



Power radiated $= e\sigma A T^4$

Power absorbed $= e\sigma A T_0^4$

$$\boxed{\text{Net power Radiated} = e\sigma A (T^4 - T_0^4)}$$

(ADV 2015)

Q. Star – A   Star – B

$R_A = 400 R_B$ and $P_A = 10^4 P_B$.

Find (λ_A/λ_B). (λ → Max. probable wavelength) sol. $P_A = P_B \times 10^4 \Rightarrow R_A^2 \times T_A^4 = R_B^2 \times T_B^4 \times 10^4$

$\Rightarrow 16 \times 10^4 \times T_A^4 = T_B^4 \times 10^4$

$\Rightarrow \frac{T_B}{T_A} = 2 = \frac{\lambda_A}{\lambda_B}$

NEWTON'S LAW OF COOLING :-

Decreases with time $\leftarrow$ Rate of cooling $\propto \Delta T$ (Temp. difference b/w body and surrounding)

$\Rightarrow \frac{\text{Fall in temp.}}{\text{time}} \propto \Delta T$

$\Rightarrow \boxed{\frac{dT}{dt} \propto T_b - T_s}$

Q. Hot water in the container

$70°C \rightarrow 65°C$; $65° \rightarrow 60°C$; $60°C \rightarrow 55°C$

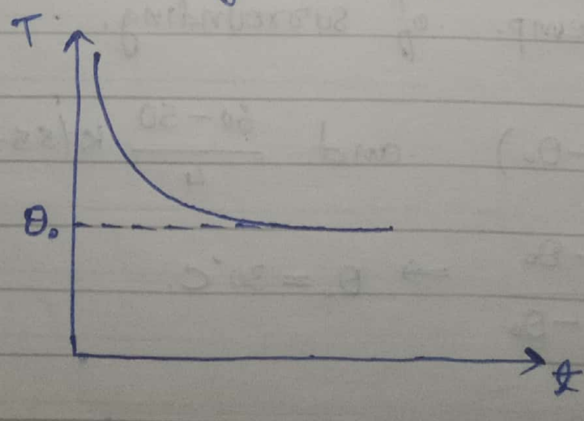
(t_1) (t_2) (t_3)

sol. As rate of cooling decreases with time ... $t_1 < t_2 < t_3$.

(MAINS 2013)

Q. A piece of metal is heated to temp. θ and kept in surrounding of temp. θ_0. The nearly correct graph for temp. of metal (T) v/s time (t) is ___.

sol. $\frac{dT}{dt}$ should be decreasing and final temp. θ_0.



⇒ **Problem solving technique :-** (Approx method) Not 100% correct

$$\theta_i \xrightarrow{\Delta t} \theta_f$$

θ_s

Variable all the time
⇒ Putting avg $= \frac{\theta_i+\theta_f}{2}$
(PHYSICS से आई)

Rate of cooling $= \frac{\text{Fall in temp.}}{\text{time taken}} = \frac{\theta_i-\theta_f}{\Delta t} \propto T_b - T_s$

$$\Rightarrow \boxed{\frac{\theta_i-\theta_f}{\Delta t} = k\left(\frac{\theta_i+\theta_f}{2} - \theta_s\right)}$$

Q. A liquid cools from 70°C to 60°C in 5 minutes. Calculate the time in which it cools from 60°C to 50°C if temp of surrounding is 30°C.

Sol. $\frac{70-60}{5} = k(65-30)$ and $\frac{60-50}{t} = k(55-30)$

$\Rightarrow 2 = 35k$ — ① and $\frac{10}{t} = 25k$ — ②

①/② ... $t = 7$ min

Q. 80°C → 60°C (5 min)
60°C → 50°C (4 min)
Find the temp. of surrounding.

Sol. $\frac{80-60}{5} = k(70-\theta_0)$ and $\frac{60-50}{4}$ $k(55-\theta_0)$

$\Rightarrow \frac{20/5}{10/4} = \frac{70-\theta_0}{55-\theta_0}$ → $\theta_0 = 30°C$.

⇒ Derivation of newton's law of cooling :-

Rate of heat loss $= e\sigma A(T^4 - T_0^4)$
(Power radiated)

$$\Rightarrow \frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$$

$$\Rightarrow \frac{mc\,dT}{dt} = e\sigma A(T^4 - T_0^4)$$

$$\Rightarrow \frac{dT}{dt} = \frac{e\sigma A}{mc}(T^4 - T_0^4)$$

$$\Rightarrow \frac{dT}{dt} = k(T^4 - T_0^4) \qquad \left(k = \frac{e\sigma A}{mc} = \text{constant}\right)$$

$$\Rightarrow \frac{dT}{dt} = k\left((T_0 + \Delta T)^4 - T_0^4\right)$$

(where; $\Delta T \to$ diff of temp. b/w body and surrounding)

$$\Rightarrow \frac{dT}{dt} = kT_0^4\left(\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right)$$

$$\Rightarrow \frac{dT}{dt} = kT_0^4\left(1 + \frac{4\Delta T}{T_0} - 1\right) \qquad [\text{Assuming } \Delta T <<< T_0]$$

$$\Rightarrow \frac{dT}{dt} = 4kT_0^3 \cdot \Delta T$$

$$\Rightarrow \frac{dT}{dt} = k'\Delta T \Rightarrow \frac{dT}{dt} \propto \Delta T$$ Proved.